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**FINAL REPORT**

**STUDY OF HIGH SPEED COMPLEX NUMBER ALGORITHMS**

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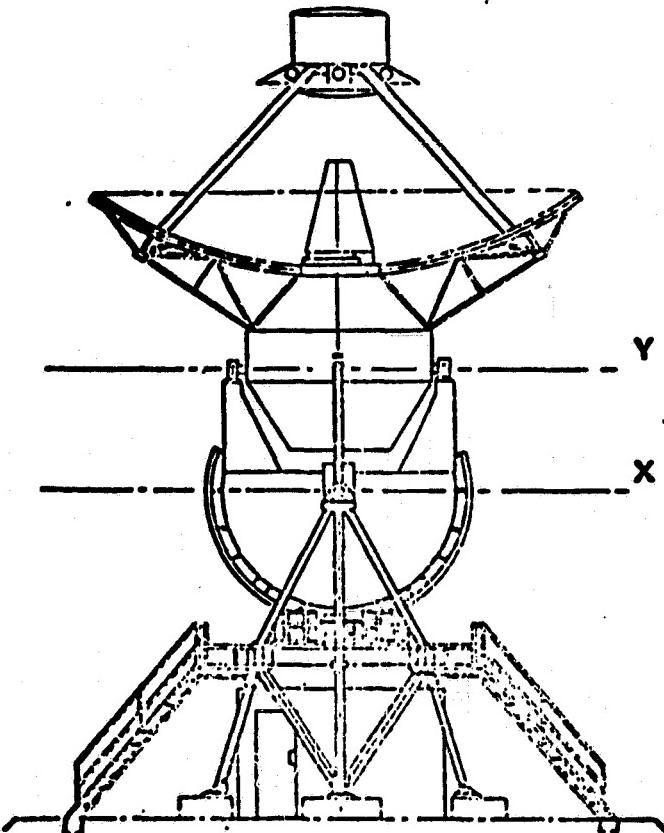
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## I. INTRODUCTION

Basic to the design of reflecting antenna systems is the need to compute the far-field radiation pattern for a given antenna geometry, feed placement, and feed radiation characteristic. A simple, yet fast solution to this classic problem has proven to be a formidable challenge. This is especially true for very large reflectors with extensive current distributions and off-focus feeds with their associated asymmetry. These problems are made troublesome because of their large computational requirements.

Recent interest in this problem [1-4] has resulted in the new application of powerful mathematical techniques to produce a more time-efficient means of computing the radiation intensity. Galindo-Israel and Mittra [1] have reformulated the two-dimensional radiation integral to a double Fourier integral "representation." The Fourier integral is evaluated by expanding the integrand in a double summation of circular functions and modified Jacobi polynomials. The result is a rapidly convergent series representation of the radiation intensity, the zeroth term being  $J_1(x)/x$ , the pattern for a uniform, cophase aperture. Mittra, et al. [2] extend the work of the first paper to offset paraboloid reflectors and introduce a more rigorous corrective technique to compensate for the non-Fourier structure of the radiation integral.

This paper presents a new method which evaluates the radiation integral on the antenna surface. The algorithm is computationally efficient and produces the far-field radiation pattern along planer cuts at any angle

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## I. INTRODUCTION

Basic to the design of reflecting antenna systems is the need to compute the far-field radiation pattern for a given antenna geometry, feed placement, and feed radiation characteristic. A simple, yet fast solution to this classic problem has proven to be a formidable challenge. This is especially true for very large reflectors with extensive current distributions and off-focus feeds with their associated asymmetry. These problems are made troublesome because of their large computational requirements.

Recent interest in this problem [1-4] has resulted in the new application of powerful mathematical techniques to produce a more time-efficient means of computing the radiation intensity. Galindo-Israel and Mittra [1] have reformulated the two-dimensional radiation integral to a double Fourier integral "representation." The Fourier integral is evaluated by expanding the integrand in a double summation of circular functions and modified Jacobi polynomials. The result is a rapidly convergent series representation of the radiation intensity, the zeroth term being  $J_1(x)/x$ , the pattern for a uniform, cophase aperture. Mittra, et al. [2] extend the work of the first paper to offset paraboloid reflectors and introduce a more rigorous corrective technique to compensate for the non-Fourier structure of the radiation integral.

This paper presents a new method which evaluates the radiation integral on the antenna surface. The algorithm is computationally efficient and produces the far-field radiation pattern along planer cuts at any angle

through the three-dimensional pattern. The physical optics approximation is used to compute the induced surface current which is the input to the algorithm. The method is developed for focal-point and translated feeds and is easily extended to offset antennas and arbitrary surfaces.

On the premise that simplification and efficiency may come from first generalizing, the radiation integral is reformulated to three dimensions. The result is shown to be a triple Fourier integral. To evaluate this integral, an extremely fast algorithm is introduced which evaluates, along planes of constant  $\phi$ , a subset of the total three-dimensional Fourier transform results. No approximations are made other than those normally associated with digitization and the discrete Fourier transform (DFT). To further reduce the computation time, the Winograd Fourier transform algorithm (WFT) is used in place of the standard radix-2 FFT when a DFT is called for in the algorithm. The any  $\phi$  angle feature of the program is implemented using a technique similar to one used in computerized tomography (cross-sectional x-rays) [5].

While other methods exist for evaluating the radiation integral, this new theory brings a fast, simple and direct approach. Due to its speed it is especially useful for very large asymmetric antennas.

## II. THE THREE-DIMENSIONAL RADIATION INTEGRAL AS A FOURIER TRANSFORM

The far-field radiation intensity of a volume distribution of current may be expressed in terms of the three-dimensional radiation integral

$$\vec{E}(\theta, \phi) = \iiint \vec{K}(r', \theta', \phi') e^{-jk(r' - \vec{r}' \cdot \hat{R})} dv' \quad (1)$$

The geometry for (1) is given in Figure 1. In formulating this equation, it is assumed that the phase center is at the origin, the observation point is far from the current distribution and the currents are bounded in a region dimensionally small compared to  $R$ .

The  $-\hat{r}' \cdot \hat{R}$  ( $\hat{R}$  a unit vector) term is the distance from the source point to a plane through the origin and normal to  $\hat{R}$ . Hence, the exponent  $k(r' - \hat{r}' \cdot \hat{R})$  is the total phase delay modulo  $2\pi R$ .

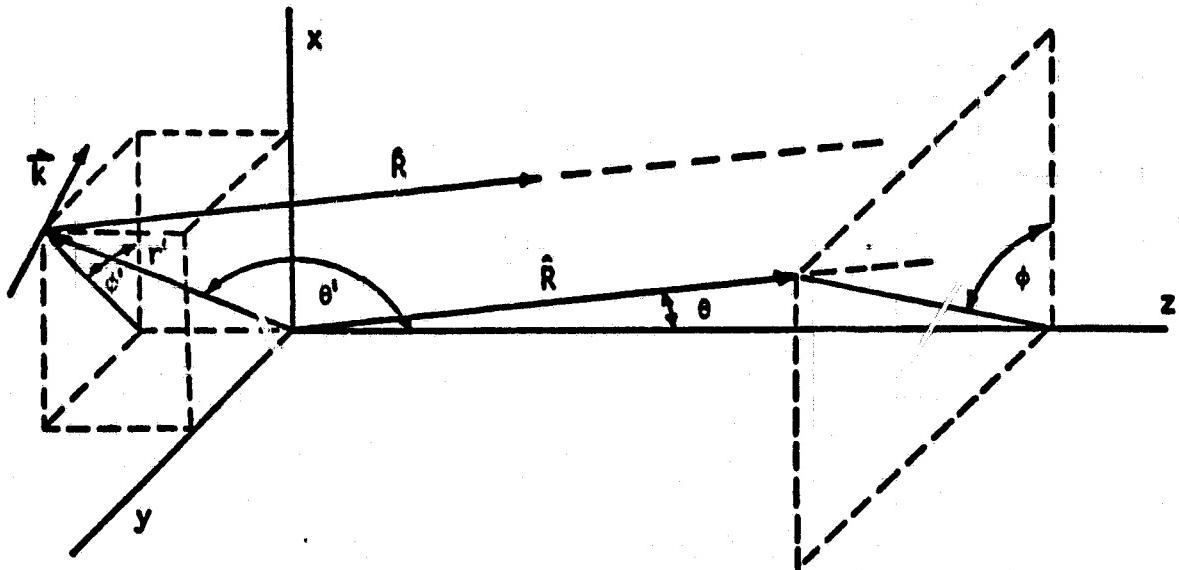


Figure 1. Geometry for the radiation integral

Using the direction cosines  $u$ ,  $v$ , and  $w$ , we can write

$$\hat{r}' \cdot \hat{R} = r'_z w + r'_x u + r'_y v$$

$$\begin{aligned}
 &= r' \cos \theta' \cos \theta + r' \cos \phi' \sin \theta' \cos \phi \sin \theta + r' \sin \phi' \sin \theta' \sin \phi \sin \theta \\
 &= r' \cos \theta' \cos \theta + r' \sin \theta' \sin \theta \cos(\phi - \phi')
 \end{aligned}$$

The radiation integral of (1) may now be rewritten as

$$\hat{E}(r, \phi) = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r'=0}^{\infty} \hat{K}(r', \theta', \phi') e^{-jkr'} e^{jk\theta' \cos\theta' \cos\phi} \\ \times e^{jk\theta' [\sin\theta' \sin\phi \cos(\phi-\phi')]} r'^2 \sin\theta' dr' d\theta' d\phi'$$
(2)

Equation (2) is a three-dimensional Fourier transform in spherical coordinates as presented by Bracewell [6]. An important corollary to this observation is that the far-field radiation pattern may be computed as the three-dimensional Fourier transform of a volume current distribution. Techniques will be presented for efficiently evaluating equation (2).

If the current exists only on a surface within the volume, the volume current function may be expressed in terms of a surface current as

$$\hat{K}(r', \theta', \phi') = \hat{J}(\theta', \phi') \delta(r' - \rho)$$

where  $\rho = \rho(\theta', \phi')$  defines the geometry of the surface. Substituting this expression into (2) and integrating with respect to  $r'$  (with the aid of the sifting property of the Dirac function) we arrive at

$$\hat{E}(r, \phi) = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \hat{J}(\theta', \phi') e^{-jk\rho} e^{jk\rho \cos\theta' \cos\phi} \\ \times e^{jk\rho [\sin\theta' \sin\phi \cos(\phi-\phi')]} r^2 \sin\theta' d\theta' d\phi'$$
(3)

Equation (3) agrees with equation (5) of Galindo-Israel [1]. To phrase it in exactly the same form requires two modifications. First, the phase center must be moved from the origin to a point defined by the feed position vector  $\vec{e}$ . Secondly, the integration must be transferred from the reflector surface to the aperture plane. The latter is accomplished by use of the "Jacobian of the transformation" between the reflector surface and the projected aperture. A single component of  $\hat{J}(\theta', \phi')$ , say  $J_x(\theta', \phi')$ , may be transformed to an "equivalent" aperture distribution

over the circular disk of the aperture. This results in the transformation

$$\begin{aligned} J_x(\theta', \phi') s^2 \sin \theta' d\theta' d\phi' &= J_x(\theta', \phi') / \sqrt{1 + (dz/ds)^2} s ds d\phi' \\ &= f(s, \phi') s ds d\phi' \end{aligned}$$

$f(s, \phi')$  is the equivalent aperture distribution and is not to be confused with the physical distribution on the aperture such as might be derived approximately by the ray technique.

Equation (3) now becomes

$$\begin{aligned} E(\theta, \phi) &= \int_{\phi=0}^{2\pi} \int_{s=0}^{\hat{u}} f(s, \phi') e^{-jkp} e^{jkp \cos \theta' \cos \theta} \\ &\quad \times e^{jkp [\sin \theta' \sin \phi \cos(\phi - \phi')]} s ds d\phi' \end{aligned} \quad (4)$$

which is the same as (5) in Galindo-Israel [1] except for a difference in phase center. Equation (4) is not a Fourier transform as  $\exp(jkp \cos \theta' \cos \theta)$  is a function of both source and observation coordinates.

### III. AN EFFICIENT ALGORITHM FOR THE THREE-DIMENSIONAL DISCRETE FOURIER TRANSFORM

The three-dimensional Fourier transform of a volume source is defined by Bracewell [6] as

$$F(u, v, w) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) e^{-j2\pi(ux + vy + wz)} dx dy dz \quad (5)$$

where  $u$ ,  $v$ , and  $w$  are the direction cosines

$$\begin{aligned} u &= \sin \theta \cos \phi \\ v &= \sin \theta \sin \phi \\ w &= \cos \theta \end{aligned} \quad (6)$$

Note that while there appear to be three independent variables in this transform, i.e., the frequencies  $u$ ,  $v$ , and  $w$ , there are in fact only two geometric variables,  $\theta$  and  $\phi$ . These may be solved for as

$$\begin{aligned} \theta &= \sin^{-1} \sqrt{u^2 + v^2} = \cos^{-1} w \\ \phi &= \tan^{-1} v/u \end{aligned} \quad (7)$$

Similarly, the three-dimensional discrete Fourier transform (DFT) is given by the triple summation

$$F(k_3, k_2, k_1) = \sum_{n_2=0}^{N_2-1} \sum_{n_1=0}^{N_1-1} \sum_{n_3=0}^{N_3-1} f(n_3, n_2, n_1) e^{-j\frac{2\pi}{N_3} n_3 k_3} e^{-j\frac{2\pi}{N_1} n_1 k_1} e^{-j\frac{2\pi}{N_2} n_2 k_2} \quad (8)$$

for  $k_1 = 0, 1, 2, \dots, N_1-1$

$k_2 = 0, 1, 2, \dots, N_2-1$

$k_3 = 0, 1, 2, \dots, N_3-1$

This procedure may be broken into three operations by the use of intermediate results. Consider the intermediate transforms

$$g(k_3, n_2, n_1) = \sum_{n_3=0}^{N_3-1} f(n_3, n_2, n_1) e^{-j\frac{2\pi}{N_3} n_3 k_3} \quad (9)$$

$n_1 = 0, 1, 2, \dots, N_1-1$

$n_2 = 0, 1, 2, \dots, N_2-1$

$k_3 = 0, 1, 2, \dots, N_3-1$

$$h(k_3, n_2, k_1) = \sum_{n_1=0}^{N_1-1} g(k_3, n_2, n_1) e^{-j\frac{2\pi}{N_1} n_1 k_1} \quad (10)$$

$k_1 = 0, 1, 2, \dots, N_1-1$

$n_2 = 0, 1, 2, \dots, N_2-1$

$k_3 = 0, 1, 2, \dots, N_3-1$

From these the DFT is found to be

$$F(k_3, k_2, k_1) = \sum_{n_2=0}^{N_2-1} h(k_3, n_2, k_1) e^{-j \frac{2\pi}{N_2} n_2 k_2} \quad (11)$$

$$k_1 = 0, 1, 2, \dots, N_1-1$$

$$k_2 = 0, 1, 2, \dots, N_2-1$$

$$k_3 = 0, 1, 2, \dots, N_3-1$$

Comparing the frequency variables of the Fourier integral and the DFT, we note the correspondence

$$u \sim \frac{k_1}{N_1 T_1}, \quad v \sim \frac{k_2}{N_2 T_2}, \quad w \sim \frac{k_3}{N_3 T_3} \quad (12)$$

where each  $T$  is the sampling period for the respective axes. Combining (12) and (7), we arrive at expressions for  $\theta$  and  $\phi$  in terms of the frequency indices of the DFT

$$\theta = \sin^{-1} \sqrt{\left(\frac{k_1}{N_1 T_1}\right)^2 + \left(\frac{k_2}{N_2 T_2}\right)^2} = \cos^{-1} \left(\frac{k_3}{N_3 T_3}\right) \quad (13)$$

$$\phi = \tan^{-1} \left( \frac{k_2}{N_2 T_2} \cdot \frac{N_1 T_1}{k_1} \right)$$

A direct evaluation of the DFT in (8) is impractical, even for modest transform lengths. Nor does use of an FFT algorithm provide sufficient improvement. However, the computation time may be reduced by orders of magnitude in the case where one is interested in a limited amount of the total DFT information. It is possible in this way to compute a two-dimensional planer cut through the far-field radiation pattern with no compromise in accuracy. Once the algorithm is developed, it may be extended to include an any angle planer cut.

The simplification is begun by selecting  $k_1 = 0$ . From equation (13) this requires that  $\phi = 90^\circ$ . Next, the value of  $k_2$  is chosen. From (13)

this determines the polar angle  $\theta = \sin^{-1}(k_2/N_2 T_2)$  and forces  $k_3$  to a specific value given by

$$\left(\frac{k_1}{N_3 T_3}\right)^2 + \left(\frac{k_2}{N_2 T_2}\right)^2 = 1 \quad (14)$$

Clearly, there are going to be digitization problems in evaluating (14) for  $k_3$  based on a selected integer value of  $k_2$ . This works to our advantage in decreasing the computation time.

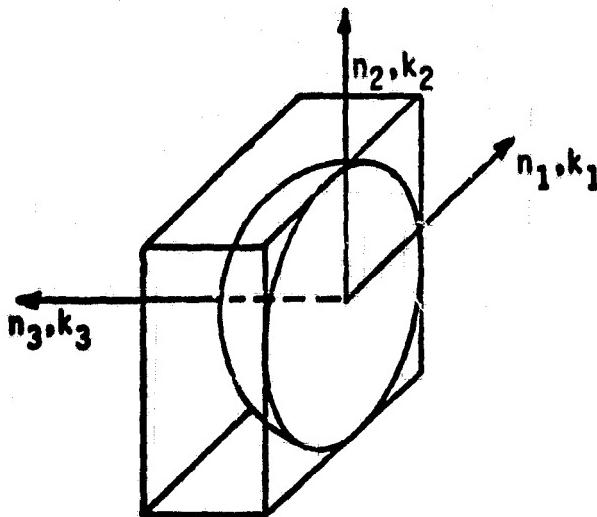


Figure 2. Antenna Geometry for the DFT

The summation in equation (9) may now be reduced.  $k_3$  is a determined single value and along the  $n_3$  dimension (see Figure 2) there is only one non-zero data element for each  $(n_1, n_2)$  pair. Hence (9) becomes

$$g(k_3, n_2, n_1) = f(n_3, n_2, n_1) e^{-j \frac{2\pi}{N_3} n_3 k_3} \quad (15)$$

$$n_1 = 0, 1, 2, \dots, N_1 - 1$$

$$n_2 = 0, 1, 2, \dots, N_2 - 1$$

$k_3$  a constant

Equation (15) describes a coalescing of the data to a planer array and hence a reduction to two-dimensions as shown in Figure 3. Each point in the  $n_1, n_2$  plane corresponds to a  $g(k_3, n_2, n_1)$ . The  $f(n_3, n_2, n_1)$ , (i.e. the current density) data will be present in polar form so that the complex multiplications indicated here are simple additions of  $\frac{2\pi}{N_3}n_3k_3$  to the phase of  $f(n_3, n_2, n_1)$ . Furthermore, the phase term  $\frac{2\pi}{N_3}n_3k_3$  need be calculated only once for each  $n_3$  and recalled as a linear array variable for the individual phase additions.

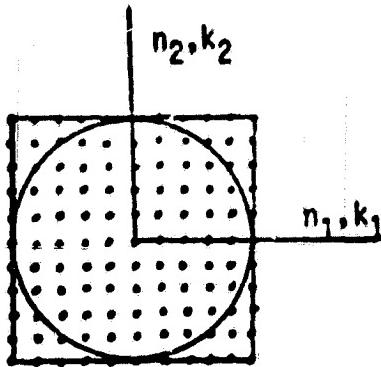


Figure 3. The coalesced two-dimensional array

With the selection of  $k_1 = 0$  comes the simplification of equation (10) to

$$h(k_3, n_2, 0) = \sum_{n_1=0}^{N_1-1} g(k_3, n_2, n_1) \quad (16)$$

$$n_2 = 0, 1, 2, \dots, N_2-1$$

$k_3$  a constant

Equation (16) describes the coalescing of a two-dimensional array to one dimension. The  $g(k_3, n_2, n_1)$  data along each row are summed to form the linear array  $h(k_3, n_2, 0)$  in  $n_2$ .

The final operation is described by equation (11)

$$F(k_3, k_2, 0) = \sum_{n_2=0}^{N_2-1} h(k_3, n_2, 0) e^{-j \frac{2\pi}{N_2} n_2 k_2} \quad (17)$$

$$k_2 = N_2 T_2 \sin \theta$$

$$k_3 = N_3 T_3 \sqrt{1 - (k_2/N_2 T_2)^2}$$

Note that (17) is not a DFT since the sum is performed for only one value of  $k_2$ . It is in fact only one of the  $N_2$  harmonic constituents of the DFT corresponding to one value of  $\theta$  in the spectrum. In practice, one may want to compute the DFT of  $h(k_3, n_2, 0)$ , as digitization problems in evaluating  $k_3$  make (17) applicable for many values of  $\theta$  (or  $k_2$ ) based on a single value of  $k_3$ .

Generally, it is not necessary to execute the above process for more than several values of  $k_3$ . For  $NT = (2520)(1/8)$  in equation (14),  $k_3 = 315$  for  $k_2$  in the range of zero to 17. This requires that for  $\theta$  between zero and  $3.1^\circ$ , the generation of the  $h(k_3, n_2, 0)$  one-dimensional array by coalescing to a plane and thence to a line need be done only once. Radiation data at  $\theta$  increments within this bound are found by evaluating (17) for  $k_2$  between zero and 17. This may be done by computing the FFT or WFT [7] of  $h(k_3, n_2, 0)$  and using only the first 18 transform results. Subsequent integer values of  $k_3$  yield  $\theta$  bounds of  $3.1^\circ$  to  $5.5^\circ$ ,  $5.5^\circ$  to  $7.1^\circ$ , etc.

The three-dimensional DFT produces a spectrum in the three variables  $k_1, k_2$ , and  $k_3$ . Such a thing is difficult to represent graphically and fortunately, in this case it is not necessary as  $k_1 = 0$ . What results, then, is a two-dimensional spectrum or a surface in  $k_2$  and  $k_3$ . Only a portion of this surface is of interest; that section defined by the elliptic relationship between  $k_2$  and  $k_3$  in equation (14). It is the spectrum

along this elliptic arc (see Figure 4) that describes the far-field radiation pattern for a constant  $\phi$  cut.

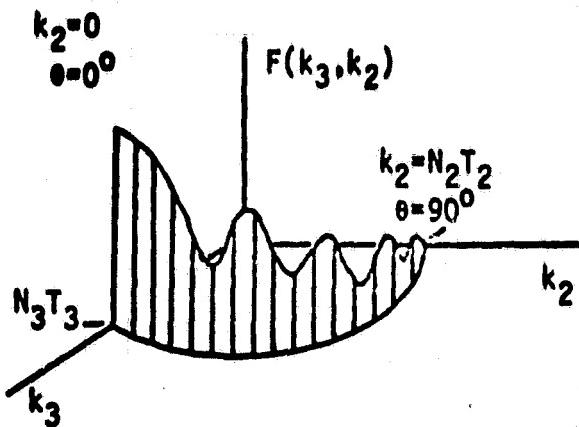


Figure 4. Two-dimensional spectrum for  $(\frac{k_2}{N_2 T_2})^2 + (\frac{k_3}{N_3 T_3})^2 = 1$

The DFT described in (8) is a "one-sided" transformation, i.e., it operates on data sequences in positive time or position only. Furthermore, the DFT is cyclic and requires that the input data be periodic. The first condition leads to a necessary format of data input. The second requires a large number of zeros to be embedded in the data to adequately isolate the antenna from distant images regularly spaced three-dimensionally about it. The geometric association of data to the  $(n_1, n_2, n_3)$  indices is demonstrated in Figure 5. The location of the origin for the  $n_1$  and  $n_2$  axes is along the center line of the antenna and is a clear extension from two-dimensional DFT theory. There is no obvious point of symmetry along the  $n_3$  axis to fix the three-dimensional origin. Consequently, it would be nice to discover that the choice did not affect the DFT results and could be made arbitrarily. It will be shown that this is true for the amplitude results, but not the phase.

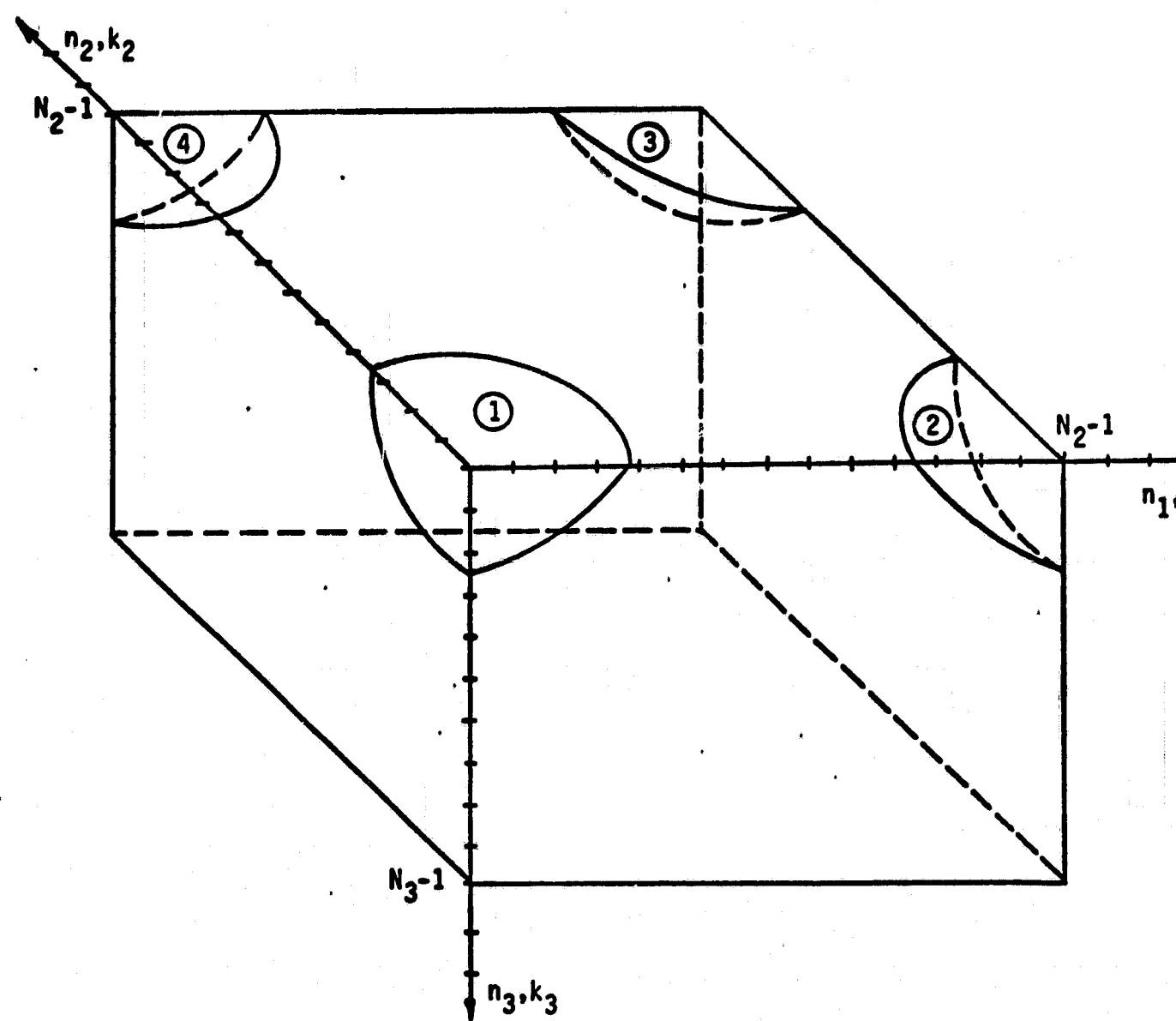


Figure 5. Data input format for the three-dimensional DFT

Equation (15) contains the  $n_3$  coordinate dependence of the DFT. A shift in the origin along the  $n_3$  axis by an integer amount  $\gamma$  will alter the phase term to

$$\begin{aligned} e^{-j\frac{2\pi}{N_3}k_3(n_3+\gamma)} &= e^{-j\frac{2\pi}{N_3}k_3\gamma} e^{-j\frac{2\pi}{N_3}k_3n_3} \\ &= e^{-j\beta} e^{-j\frac{2\pi}{N_3}k_3n_3} \end{aligned}$$

When this data is coalesced to a line, the resulting one-dimensional data vector will have a constant phase shift as seen from equation (16)

$$\begin{aligned} \sum_{n_1=0}^{N_1-1} f(n_3, n_2, n_1) e^{-j\frac{2\pi}{N_3}n_3 k_3} e^{-j\beta} \\ = e^{-j\beta} \sum_{n_1=0}^{N_1-1} g(k_3, n_2, n_1) \\ = e^{-j\beta} h(k_3, n_2, k_1) \end{aligned}$$

Hence, a constant phase shift,  $\beta$ , will be present in all the DFT terms, but the amplitude results remain unaltered. A constant phase shift is certainly not a problem. However, difficulty does arise when the algorithm is repeated for other values of  $k_3$ . Recall that repeated application is necessary in order to sweep through a broad range in  $\theta$  and that the number of separate values of  $k_3$  is determined by this  $\theta$  range (three applications for  $\theta$  out to  $7.1^\circ$ ). Clearly,  $\beta$  is a function of  $k_3$  and the separate composite ranges will each be shifted by a different constant phase. If phase information is important, it is an easy matter to compute these phase terms and subtract them from the results.

Finally, the direction of the  $n_3$  coordinate is important. The Fourier transform may be formulated with either a plus or minus exponential

phase term in the integrand. The  $n_3$  coordinate orientation shown in Figures 2 and 5 is consistent with the negative phase form and when used with this definition produces correct transform results.

#### IV. AN ANY ANGLE PATTERN CUT

The selection of  $k_1 = 0$  in the previous section was significant in that it allowed us to simplify the three-dimensional DFT by coalescing the planer data to a linear array. However, it also limited our consideration to the  $\phi = 90^\circ$  plane. It is imperative that we be able to examine the radiation intensity along other planer cuts through the pattern. This may be done using a projection technique similar to that used in computerized tomography [5] and multiangular scanning in gases [8].

Equation (16) indicates that the planer data (see Figure 3) be summed along rows of constant  $n_2$  to produce a one-dimensional data vector along the  $n_2$  ( $\phi=90^\circ$ ) axis. Following the same procedure, the data may be projected to a diagonal line  $n'_2$  (see Figure 6) at an angle  $\phi$  to the  $n_1$  axis. Conceptually, this is equivalent to a rotation of the coordinate system about the  $n_3$  axis. The resulting data vector is then operated on by equation (17) to produce a planer cut at the new angle  $\phi$ .

From Figure 6, the complex data are summed along grid tubes (of width T) perpendicular to the diagonal axis. Looking along these tubes, one observes that the data points do not lie in the centers of the new grid squares and that a few squares contain no date points, while others contain two. The first difficulty is related to quantitization error which is always present when converting a continuous function to discrete data. The assumption that an interval of a continuous function may be

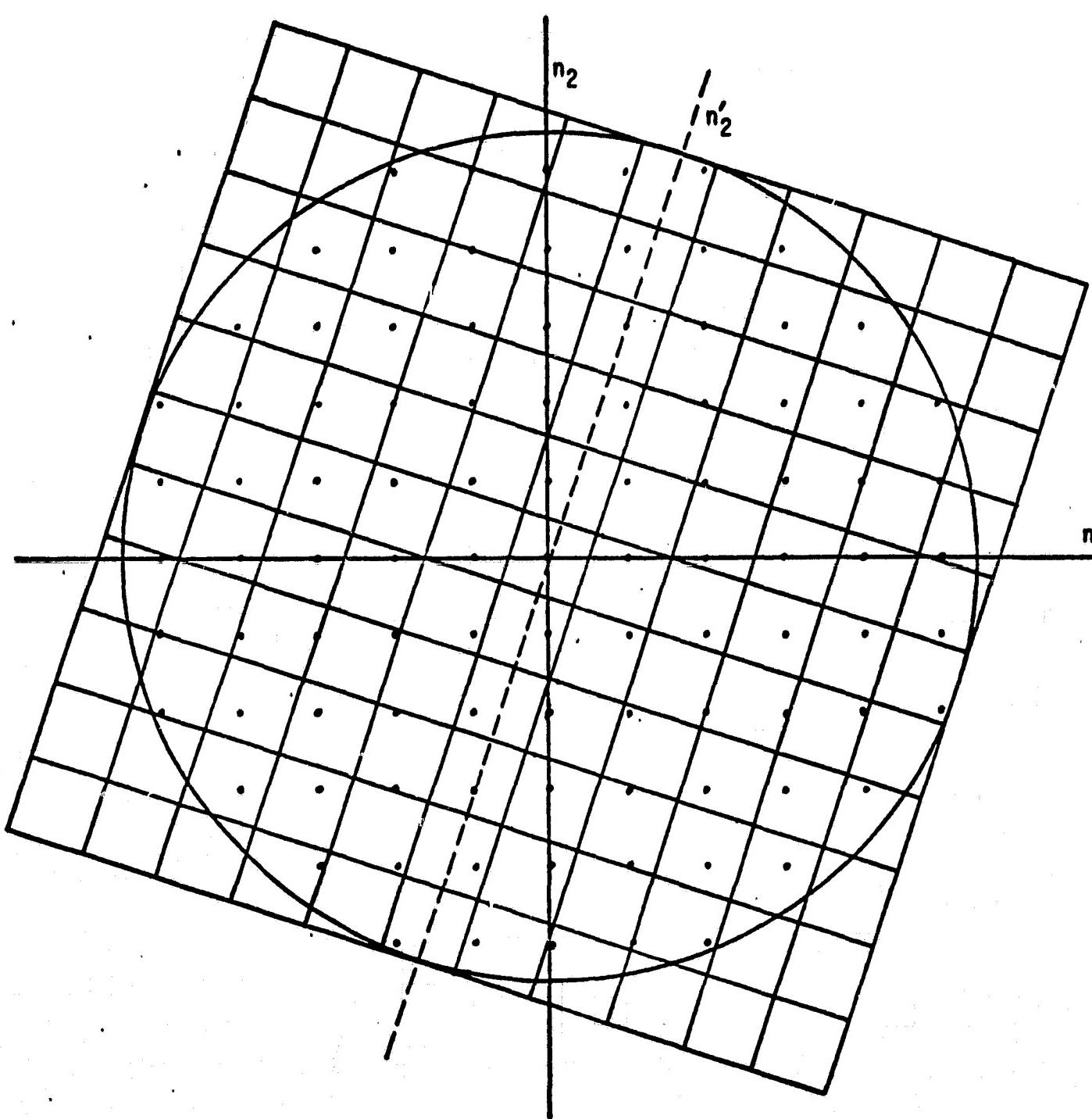


Figure 6. Coalescing to a diagonal line for an any angle cut

represented by a constant value is a first order type of approximation; that the constant is not the value of the function at the center point is second order.

As for the second difficulty, the concern here is that the sum along each grid tube be essentially the same as if resulting from a regular grid with one and only one value associated with each square. This will be true if the sampling rate is sufficient and if the sums are adjusted to account for any irregularity in the number of data in each tube.

## V. CALCULATION OF THE INDUCED ANTENNA CURRENTS

The physical optics approximation was used to compute the antenna surface currents. This comes from applying the magnetic boundary conditions

$$\vec{J} = 2\hat{n} \times \vec{H} \quad (18)$$

Generally, this is considered sufficiently accurate for studies of the main beam and several side lobes. It is only necessary to determine the unit vector ( $\hat{n}$ ) normal to the surface and the magnetic field ( $\vec{H}$ ). The currents are derived for a paraboloid reflector with a  $(-\cos\theta')^n$  offset feed and a feed displacement  $\vec{p}_e = (x_e, y_e, z_e)$ . Assuming a unit electric polarization in the y-direction, a  $1/\rho'$  space divergence of the field and a  $e^{-jk\rho'}$  phase delay, we determine

$$\begin{aligned} J_x &= \frac{-x'y/2f}{\sqrt{1 + (\sigma/2f)^2 \sqrt{x'^2 + z'^2}}} (-\cos\theta')^n \frac{e^{-jk\rho'}}{\rho'} \\ J_y &= \frac{-z' + x'x/2f}{\sqrt{1 + (\sigma/2f)^2 \sqrt{x'^2 + z'^2}}} (-\cos\theta')^n \frac{e^{-jk\rho'}}{\rho'} \\ J_z &= \frac{yz'/2f}{\sqrt{1 + (\sigma/2f)^2 \sqrt{x'^2 + z'^2}}} (-\cos\theta')^n \frac{e^{-jk\rho'}}{\rho'} \end{aligned} \quad (19)$$

The geometric variables for (19) are defined in Figure 7.

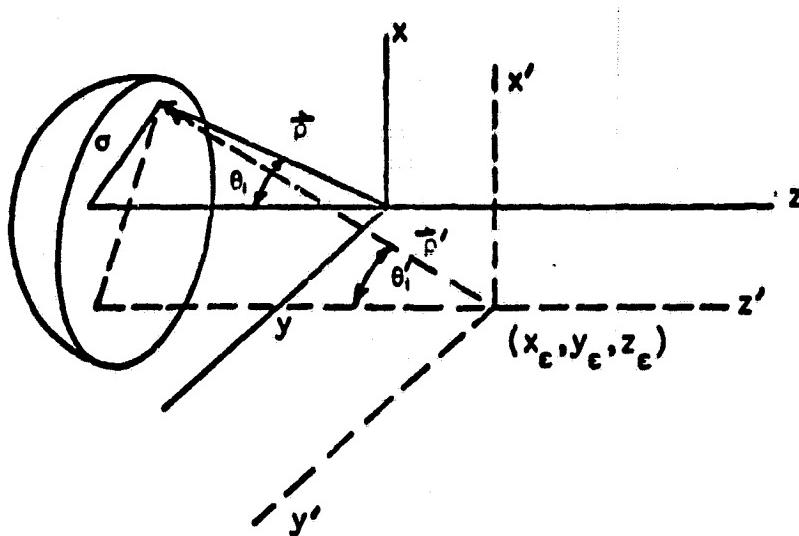


Figure 7. Geometry of focal-point and translated feeds.

## VI. COMPUTATIONAL RESULTS

Performance of the new three-dimensional algorithm is verified by comparing computed results with other published work. A driver program was used to generate, on the paraboloid, a surface current of constant amplitude. Furthermore, a phase advance  $e^{j2\pi n_3/T_3}$  was assigned to the current at each point so that the field at the aperture would be both uniform and cophase. This provided a test of the new algorithm with classical theory. The computed result was the expected  $J_1(x)/x$  (see Figure 8).

Radiation patterns were computed for currents derived by the physical-optics approximation. The reflector is characterized by an  $f/D = 0.5$ , and the feed pattern as a circularly symmetric  $(-\cos\theta)^n$ . The value of  $n$  was chosen to produce a -10dB taper to the edge of the reflector. Figures 9(a)

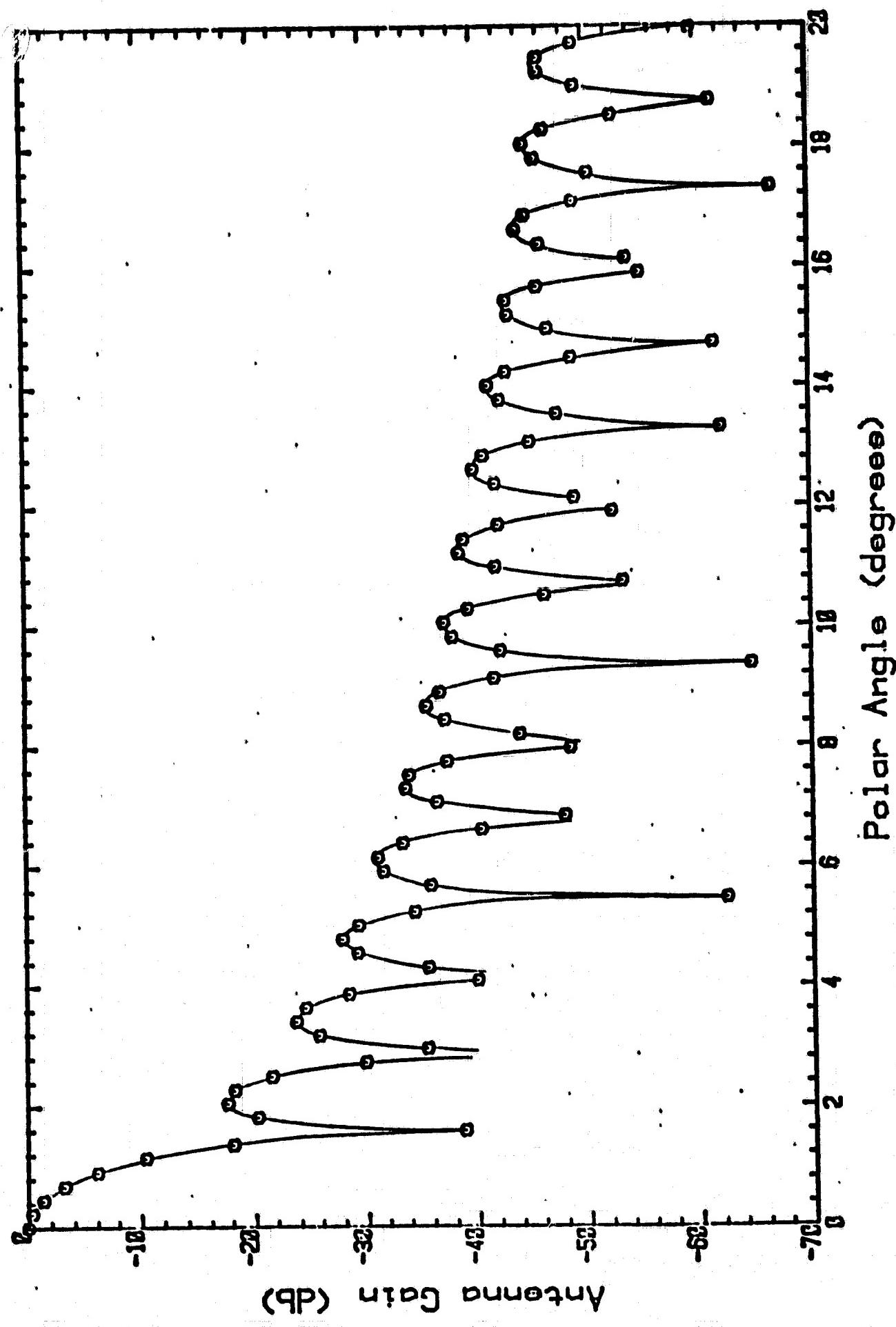


Figure 8. Radiation pattern for a uniform, cophase aperture by the three-dimensional DFT

and 10(a) present radiation patterns in the  $\phi = 90^\circ$  plane for a focal-point feed and an offset feed ( $y_e = -1.25\lambda$ ). The phase results are given in Figures 9(b) and 10(b). These results may be compared with Figures 3 and 4 of Galindo-Israel and Mittra [1] where amplitude and phase patterns were computed for similar antenna parameters using the Jacobi polynomial method. Careful examination reveals strong agreement between the two methods.

A more rigorous test of the algorithm is to translate the feed  $1.25\lambda$  along a line bisecting the  $-x$  and  $-y$  axes and compute a  $\phi = 45^\circ$  cut through the pattern. In this case, the feed displacement parameters are  $x_e = y_e = -0.883883\lambda$  and  $z_e = 0$ . The resulting pattern should be essentially the same (for  $\theta < 10^\circ$ ) as computed for  $y_e = -1.25\lambda$ ,  $x_e = z_e = 0$ , and  $\phi = 90^\circ$ . Excellent agreement is clearly seen in Figure 11. Due to polarization, the  $\phi = 45^\circ$  results fall increasingly further below the  $\phi = 90^\circ$  results as  $\theta$  becomes larger.

Figure 12 demonstrates the flexibility of this new algorithm. For a feed offset  $y_e = -1.25\lambda$ , and  $x_e = z_e = 0$ , pattern cuts are computed for  $\phi = 90^\circ$ ,  $60^\circ$ ,  $30^\circ$ , and  $0^\circ$ . Repeated application to other  $\phi$  angles can adequately describe the three-dimensional nature of the radiation pattern.

By experience, the new three-dimensional algorithm was found to be computationally very fast. Timing was done on the Goddard Space Flight Center's IBM 360/91 using the interval timer available in the system library. With a very close  $\lambda/8$  sampling, the cpu run time per pattern (for the general case) was approximately 25 seconds. A  $\lambda/4$  sampling was found to be sufficient to study the main beam and several side lobes (see Figure 13), and ran in a mere seven seconds. Circularly symmetric current sheets may be treated as a special case and consequently, run times reduced by about 75 percent. For the cpu times cited, most of the time is

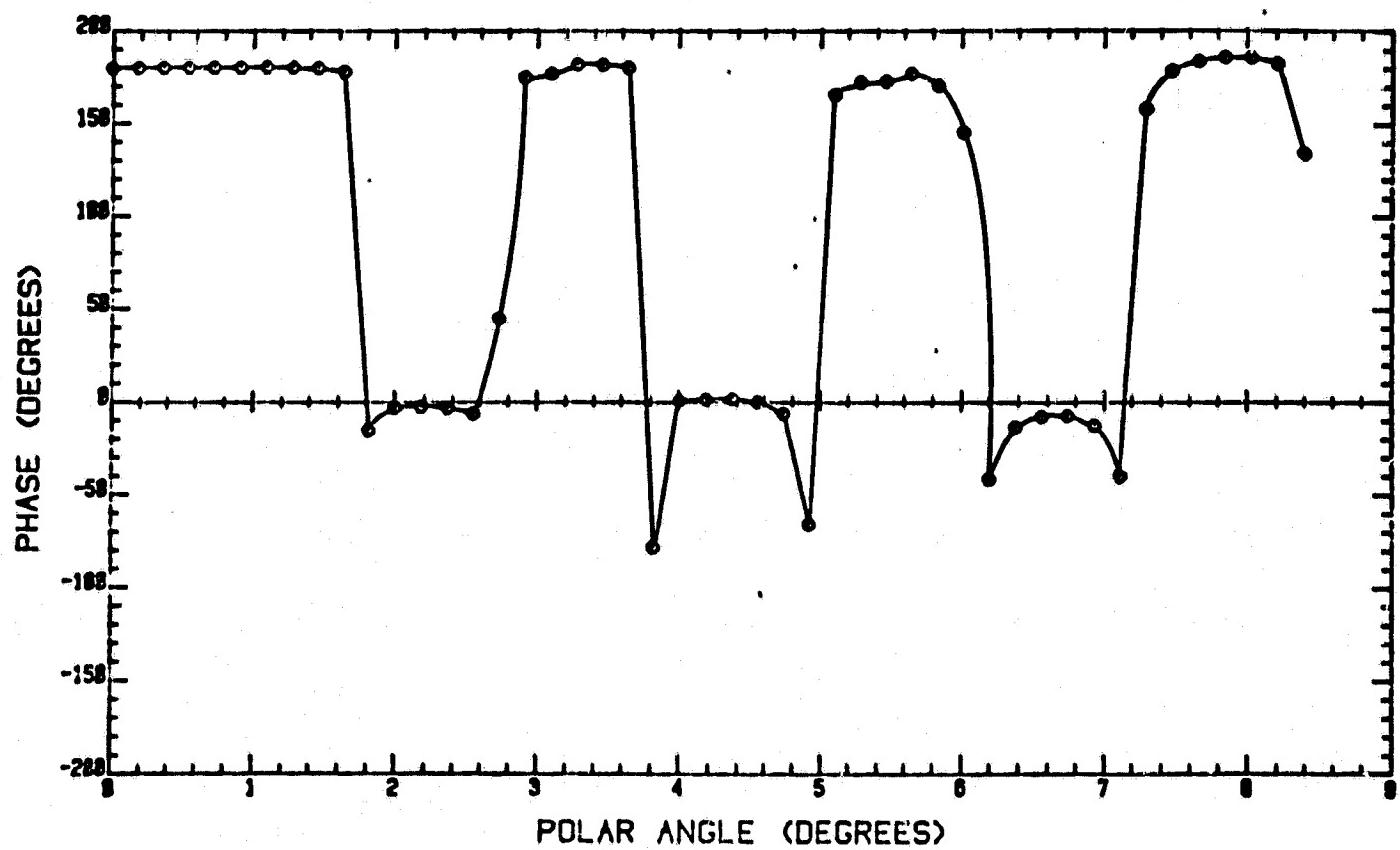
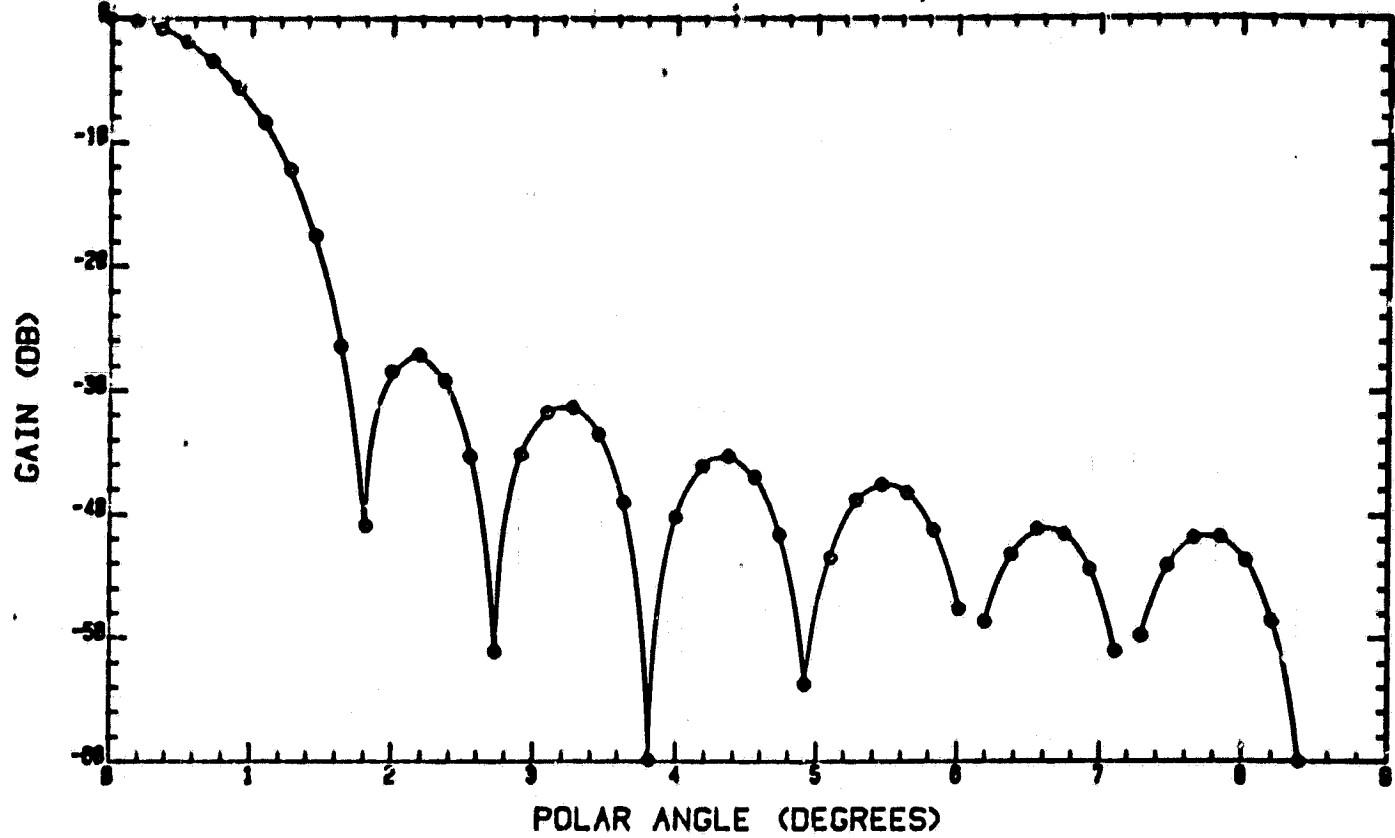


Figure 9. Gain and phase patterns for a focal-point feed.  
 $D/\lambda = 50$ ,  $f/D = 0.5$ ,  $T = \lambda/8$  and  $N = 2520$ .

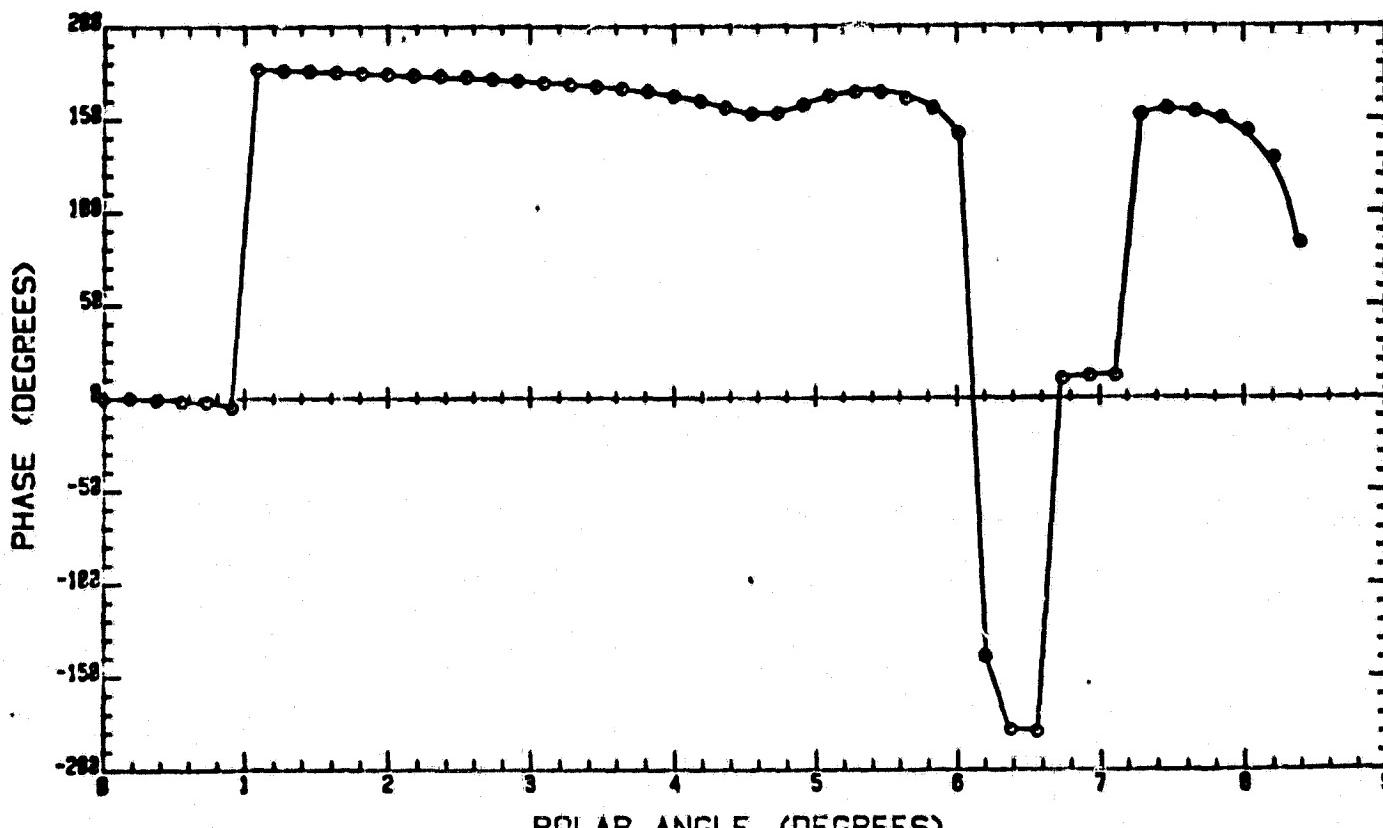
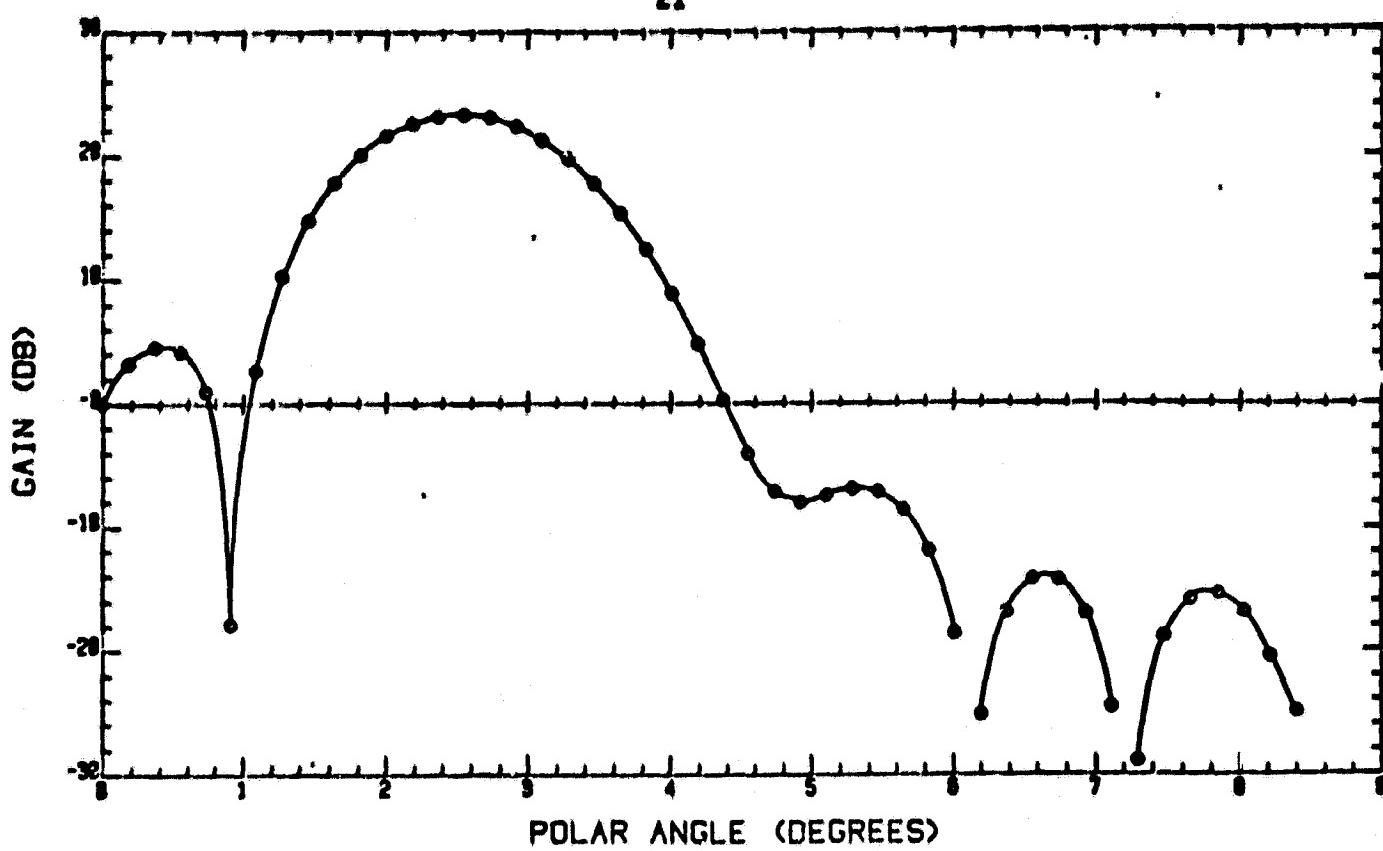


Figure 10. Gain and phase patterns for an offset feed.  
 $y_e = -1.25\lambda$ ,  $x_e = z_e = 0$ ,  $D/\lambda = 50$ ,  $f/D = 0.5$ ,  
 $T = \lambda/8$  and  $N = 2520$ .

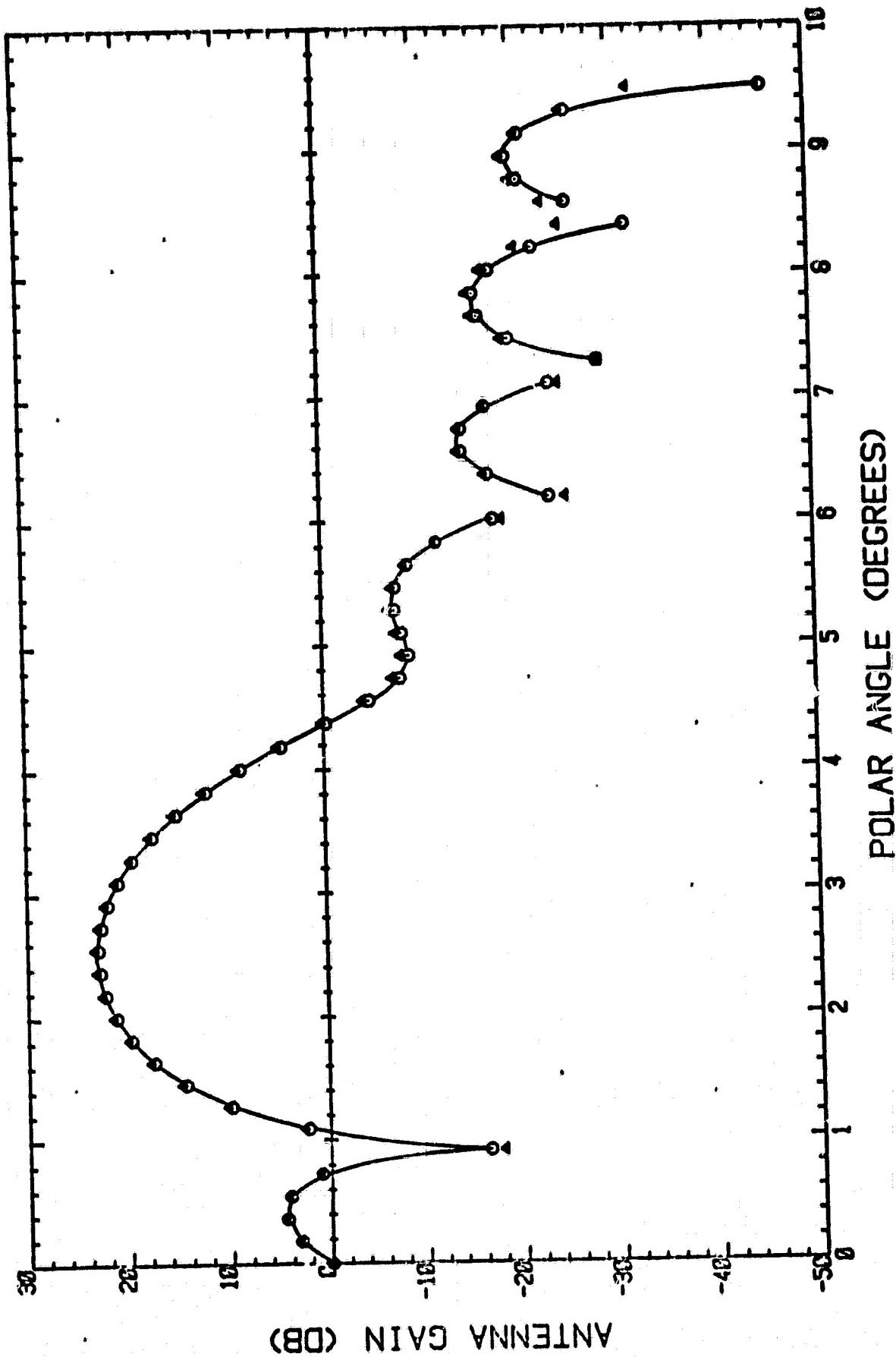


Figure 11. Diagonal axis radiation pattern for a diagonally offset feed.  
 $\circ-\circ$   $x_e = y_e = 0$ , and  $\phi = 45^\circ$ .  $\triangle$   $y_e = -0.883883\lambda$ ,  $z_e = 0$   
 $\bullet-\bullet$   $z_e = 0$ , and  $\phi = 90^\circ$ .

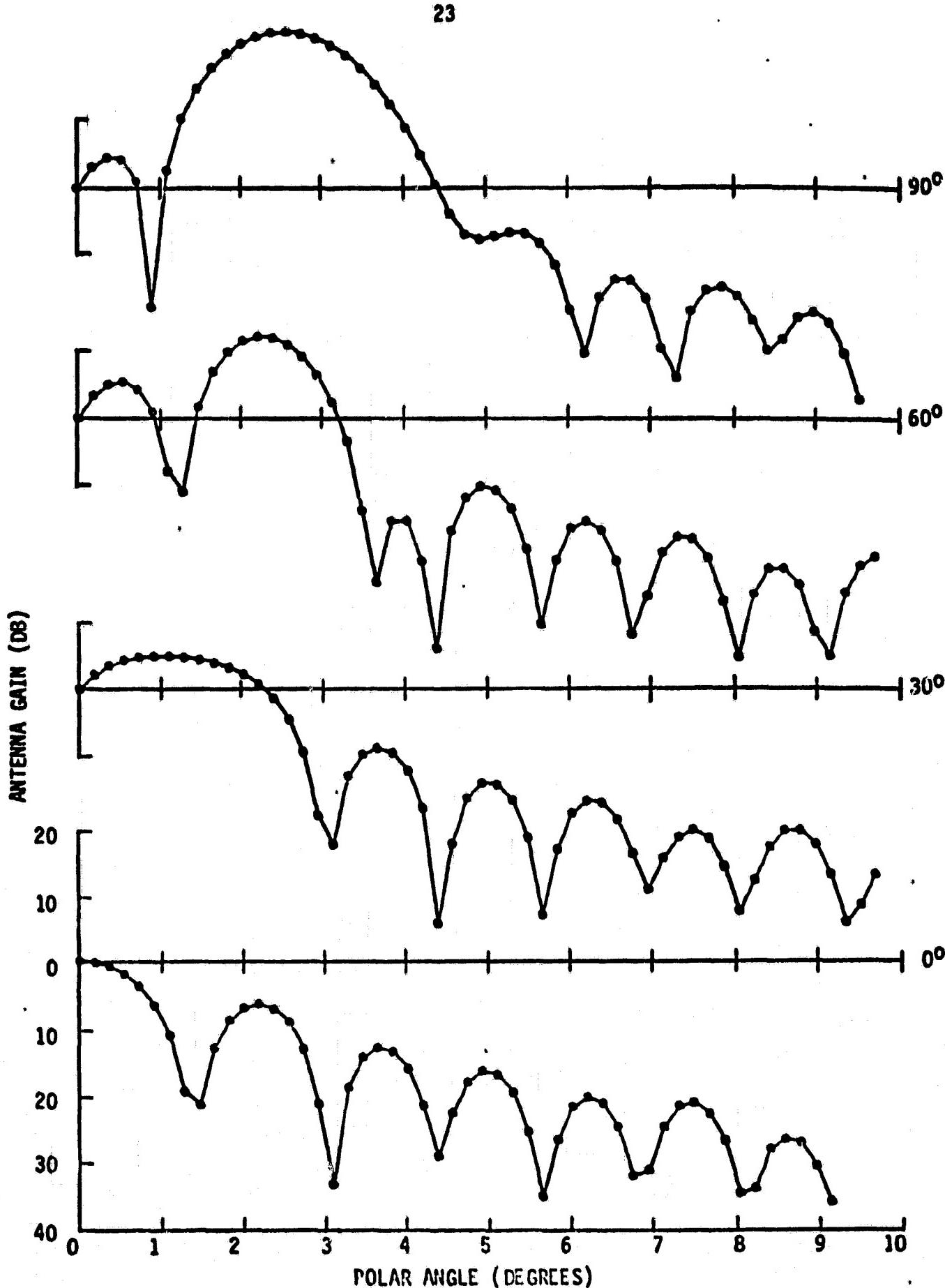


Figure 12. Selected sectional cuts through the radiation pattern.  
 $y_c = -1.25\lambda$ ,  $x_c = z_c = 0$ ,  $D/\lambda = 50$ ,  $f/D = 0.5$ ,  $T = \lambda/8$   
and  $N = 2520$ .

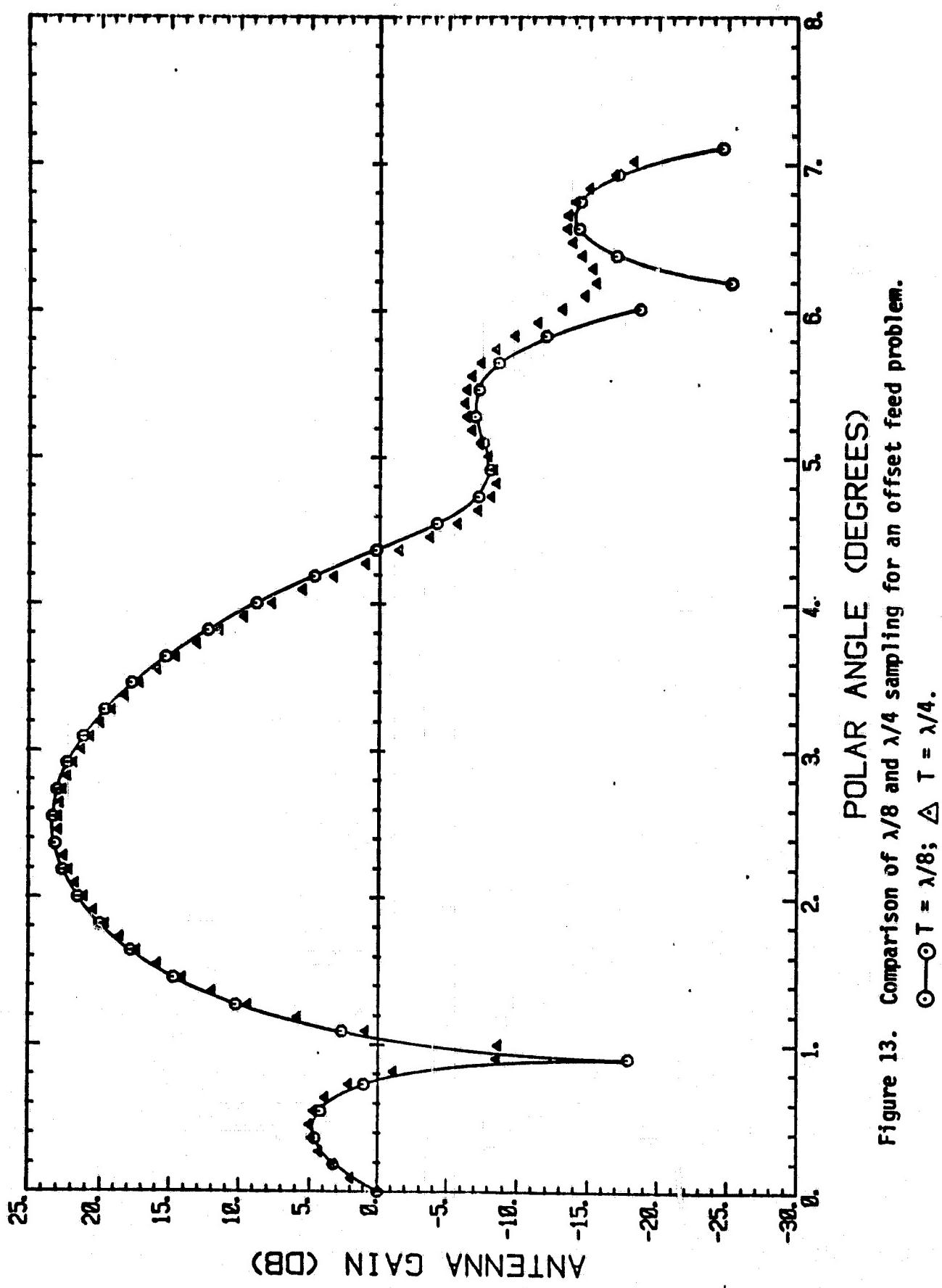


Figure 13. Comparison of  $\lambda/8$  and  $\lambda/4$  sampling for an offset feed problem.  
 $\circ - \circ T = \lambda/8; \triangle - \triangle T = \lambda/4.$

consumed in computing the induced currents. The partial three-dimensional DFT results are computed in much less than half of these times.

## VII. CONCLUSIONS

A method of evaluating the radiation integral on the curved surface of a reflecting antenna has been presented. The result is a two-dimensional radiation cross-section along a planer cut at any angle  $\phi$  through the far-field pattern. This section is produced by evaluating the radiation integral via a three-dimensional Fourier transform. A unique feature of this method is a new algorithm for evaluating a subset of the total three-dimensional DFT results. The algorithm is extremely fast so that the computer time required to produce a radiation pattern is primarily determined by the computation of the antenna currents.

High quality gain and phase results have been computed for a paraboloid reflector with translate feed. However, the method is easily extended to offset antenna systems and reflectors of arbitrary shapes. The new method provides a direct but fast approach to the analysis of large asymmetric reflector antennas.

## ACKNOWLEDGMENT

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C THIS PROGRAM COMPUTES THE SURFACE CURRENTS IN A PARABOLIC  
 C REFLECTING ANTENNA AS PRODUCED BY AN OFFSET CENTER TO THE FEED  
 C FEED AND THEN THE FAR-FIELD PATTERN BY MEANS OF A 3-D DFT.  
 C THE INPUTS ARE:  
 C DISTANCE IN WAVELENGTHS  
 C FEED/ROTATION ANGLE (IN DEG) OF THE CUT THROUGH THE PATTERN  
 C XE = X FEED OFFSET IN WAVELENGTHS  
 C YE = Y FEED OFFSET IN WAVELENGTHS  
 C ZE = Z FEED OFFSET IN WAVELENGTHS  
 C K4 IS A PARAMETER THAT SPECIFIES THE THATA LIMIT OF THE PATTERN  
 REAL\*8 A,B,SUMR,SUMR1,RATIO,C12  
 DIMENSION LIMITX(5040)

COMMON/DATA/SUMR(5040),SUMR1(5040)

COMMON/ZPAR1/M1,M2,M3,M4,LT,KNUT

COMMON/ZPAR2/M1,M2,M3,M4

COMMON/ZPAR3/I,11,X,Y,XE,YE,ZE,SUMRAZ,F,KLIMIT,PL,

1PHASES(100),ICRHT(5040)

COMMON/ZDFT/A(5040),B(5040)

COMMON/N,T

CALL RMT100(ICRH)

READ(S,100) D,FD,ANLD,XE,YE,ZE,K4

100 FORMAT(6F10.3,14)

T=0.125

XE=XE/T

YE=YE/T

ZE=ZE/T

M1=0/T+1

K3=315

WRITE(6,150)

150 FORMAT(1H),15X,25HFEED FIELD ANTENNA PATTERN/7X,5HANGLE,4X,  
 27HARG(DB),5X,5PHASE)

1 N=2520

M1=7

M2=4

M3=5

M4=8

NFT=4

KNUT=1459

RAD=1.745329E-2

PI=3.141597

ANLD=ANLD\*RAD

F=(M1-1)\*FD

KLIMIT=INT((M1-1)/(15.0\*FD)+0.5)+1

DO 3 K=1,KLIMIT

PHASES(K)=FLOAT(K-1)\*ELIMAT(K3)/N

3 CONTINUE

DO 5 I=1,N

SUMR(I)=0.000

SUMR1(I)=0.000

ICRHT(I)=0

LIMITX(I)=0

5 CONTINUE

LIMTY1=(1+N1)/2

LIMTY2=M1-(M1-1)/2

DO 200 I=1,N

IF(I.LE.LIMTY1) GO TO 10

IF(I.GE.LIMTY2) GO TO 50

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1=LINTY2-1
GO TO 200
10 Y=I-1.0
Y2=Y*Y
ARG=(LINTY1-1.0)**2-Y2
TX1=SORT(ARG)+1.0
LINTX1=TX1+0.5
LINTX2=N+2-LINTX1
LINTX(1)=LINTX1**2-1
C INPUT DATA FOR QUADRANT ONE
DO 20 J=1,LINTX1
X=J-1.0
IF(J,LE,0.1) GAMA=90.0*RAD
IF(J,GT,1) GAMA=ATAN(Y/X)
PSI=AM1-GAMA
SIGMA2=X*X+Y2
SIGMA=SORT(SIGMA2)
PROJAN=SIGMA*COS(PSI)
II=INT(Projan+0.5)+1
IF(Projan,LT,-0.5) II=N+1+INT(Projan-0.5)
CALL KCOMP
20 CONTINUE
C INPUT DATA FOR QUADRANT TWO
IF(LINTX2,GT,N) GO TO 200
DO 30 J=LINTX2,N
X=-(N-J+1.0)
IF(J,LE,0.1) GAMA=90.0*RAD
IF(J,GT,1) GAMA=ATAN(-X/Y)
PSI=90.0*RAD-AM1+GAMA
SIGMA2=X*X+Y2
SIGMA=SORT(SIGMA2)
PROJAN=SIGMA*COS(PSI)
II=INT(Projan+0.5)+1
IF(Projan,LT,-0.5) II=N+1+INT(Projan-0.5)
CALL KCOMP
30 CONTINUE
GO TO 200
50 Y=-(N+1.0-1)
Y2=Y*Y
ARG=(LINTY1-1.0)**2-Y2
TX1=SORT(ARG)+1.0
LINTX1=TX1+0.5
LINTX2=N+2-LINTX1
LINTX(1)=LINTX1**2-1
C INPUT DATA FOR QUADRANT FOUR
DO 60 J=1,LINTX1
X=J-1.0
IF(J,LE,0.1) GAMA=90.0*RAD
IF(J,GT,1) GAMA=ATAN(-Y/X)
PSI=AM1+GAMA
SIGMA2=X*X+Y2
SIGMA=SORT(SIGMA2)
PROJAN=SIGMA*COS(PSI)
II=INT(Projan+0.5)+1
IF(Projan,LT,-0.5) II=N+1+INT(Projan-0.5)+1
CALL KCOMP
60 CONTINUE
C INPUT DATA FOR QUADRANT THREE
IF(LINTX2,GT,N) GO TO 200

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DO 70 J=1,1ATX2,N
X=(M-J+1)*01
GAMA=ATAN(Y/X)
PSI=PI-GAMA
SIGMA2=X*X+Y*Y
SIGMA=SQR(SIGMA2)
PROJAN=SIGMA*COS(PSI)
II=0-INT(PROJAN)+0.51+
IF(Projan.LT.0.5) II=INT(-Projan+0.5)+1
CALL KCIMP
70 CONTINUE
200 CONTINUE
DO 250 I=1,N
IF(ICOUNT(I).EQ.00) GO TO 250
RAT=DEFDAT(LIMITX(I))/DEFDAT(ICOUNT(I))
SUMRR(I)=SUMRR(I)*RAT
SUMRI(I)=SUMRI(I)*RAT
250 CONTINUE
CALL GOODFT(N)
CALL PATOUT
KB=KB-1
IF(KB.GE.K4) GO TO 1
CALL RMT100(ICPU2)
ICPU3=ICPU-ICPU2
WRITE(6,160) ICPU3
160 FORMAT(1H1,5HICPU,1I0)
STOP
END
SUBROUTINE KCIMP
REAL RR,SUMRR,SUMRI
COMMON/DATA/SUMRR(5040),SUMRI(5040)
COMMON/PAR3/I,II,X,Y,XE,YE,ZE,Y2,SIGMA2,F,KLIMIT,PI,
1PHASE3(100),ICOUNT(5040)
COMMON N,T
F2=2.0*F
K=KLIMIT-INT(SIGMA2/(4.0*F)+0.5)
XE=X-XE
YE=Y-YE
ZE=F-KLIMIT+K+ZE
XP2=X*X*XP
YP2=Y*Y*YP
ZP2=Z*Z*ZP
RHOP=SQRT(XP2+YP2+ZP2)
CUTHP=RHP/RHOP
AKG2=SIGMA2/(F2*F2)
DEMOP=SQRT((1.0+AKG2)*(XP2+ZP2))
RHP2=ZP+XP*XE/F2
XHAG=CUTHP*(CUTHP*#2.2538)/(DEMOP*RHP*PI)
XPHASE=-2.0*PI*(PHASE3(K)+RHP*PI)
XRE=XHAG*COS(XPHASE)
XIM=XHAG*SIN(XPHASE)
SUMRR(II)=SUMRR(II)+XRE
SUMRI(II)=SUMRI(II)+XIM
ICOUNT(II)=ICOUNT(II)+1
RETURN
END
SUBROUTINE GOODFT(N)

```

C THE SUBROUTINE GOODFT COMPUTES A LENGTH N DFT OF THE INPUT DATA WHICH IS

C TWO VECTORS, XR THE REAL PART AND XI THE IMAGINARY PART. BOTH XR AND XI  
C ARE LENGTH N VECTORS. THE LENGTH OF THE DFT, N, MUST BE A PRODUCT OF AT MOST  
C FOUR MUTUALLY PRIME FACTORS. THE POSSIBLE FACTORS ARE 2, 3, 5, 7, 11, 13  
C THESE FACTORS ARE 11, 12, 13, AND 14. IF THE FOUR FACTORS ARE NOT ALL USEFUL  
C THE UNUSED FACTORS ARE SET EQUAL TO 1. FOR EXAMPLE WITH N=30, WE HAVE  
C K1=5, K2=3, K3=2, AND K4=1. THE FACTORS OF ONE MUST BE THE LAST OF THE  
C THE NUMBER OF NONUNITY FACTORS IS NFT. KOUT IS AN OUTPUT INDEXING CONSTANT  
C WHICH IS PRECOMPUTED. KOUT=(K1+K2+K3+K4)MOD N WHERE K1=M2\*M3\*M4,  
C K2=M1\*M3\*M4, K3=M1\*M2\*M4, AND K2=0 IF M2=1, K3=0 IF M3=1, AND  
C K4=0 IF M4=1. FOR EXAMPLE, N=30, K1=6, K2=10, K3=15, K4=0 AND KOUT=51=11+12+13  
C =1. THE TRANSFORMED RESULTS ARE STORED IN TWO LENGTH N VECTORS, A AND B.  
C A CONTAINS THE REAL PART AND B CONTAINS THE IMAGINARY PART OF THE RESULTS.

IMPLICIT REAL\*8 (A-H,O-Z)

COMMON/DATA/XR(5040),XI(5040)

COMMON/DET/A(5040),B(5040)

COMMON/PAR1/M1,42,M3,M4,NFT,KOUT

DIMENSION DR(16),UI(16),I(16)

REAL\*8 MR1,MR2,MR3,MR4,MR5,MR6,MR7,MR8,MR9,MR10,MR11,MR12,MR13

REAL\*8 MR14,MR15,MR16,MR17,MR18,MR19,MR20,MR21,MR22,MR23,MR24

REAL\*8 MR25,MR26,MR27,MR28,MR29,MR30,M11,M12,M13,M14,M15,M16,M17

REAL\*8 M18,M19,M110,M111,M112,M113,M114,M115,M116,M117,M118,F119

REAL\*8 M120,M121,M122,M123,M124,M125,M126,M127,M128,M129,M130

NF=NFT

C ORDER FACTORS FOR TRANSFORMS OF LENGTH M1

MM1=M1

MM2=M2

MM3=M3

MM4=M4

GO TO 20

10 GO TO 12,13,14),NF

C ORDER FACTORS FOR TRANSFORMS OF LENGTH M2

12 MM1=M2

MM2=M1

MM3=M3

MM4=M4

GO TO 20

C ORDER FACTORS FOR TRANSFORMS OF LENGTH M3

13 MM1=M3

MM2=M1

MM3=M2

MM4=M4

GO TO 20

C ORDER FACTORS FOR TRANSFORMS OF LENGTH M4

14 MM1=M4

MM2=M1

MM3=M2

MM4=M3

C INDEXING INITIALIZATION FOR THE TRANSFORMS

20 N2=0

N3=0

N4=0

K1=MM2\*MM3\*MM4

K2=MM1\*MM3\*MM4

K3=MM1\*MM2\*MM4

K4=MM1\*MM2\*MM3

I(1)=0

C INPUT INDEXING ALONG ONE DIMENSION

21 DO 22 J=2,MM1

I(J)=I(J-1)+K1

I(J)=I(J-1)+K2

I(J)=I(J-1)+K3

22 CONTINUE

C TRANSFERS THE DATA TO THE PRIMARY VECTORS DR AND DT

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30 DO 31 J=1,MM1  
  I(J)=I(J)+1  
  UR(J)=XR(J,J)  
31 UI(J,J)=XI(J,J)

C TRANSFER UR,UI  
  GU TO(50,200,300,400,500,50,700,800,400,50,50,50,50,50,50,1600),MM  
  21

C PLACE RESULTS OF TRANSFORM BACK IN XR AND XI  
40 DO 41 J=1,M11  
  I(J)=I(J)+1  
  XR(J,J)=UR(J,J)  
41 XI(J,J)=UI(J,J)

C TESTING FOR COMPLETION OF THIS FACTOR'S TRANSFORMS  
  IF(N2.NE.MM2-1) GO TO 51  
  N2=0  
  IF(N3.NE.MM3-1) GO TO 52  
  N3=0  
  IF(N4.NE.MM4-1) GO TO 53  
50 NF=MF-1  
  IF(NF.EQ.0) GO TO 1000  
  GO TO 10

C INPUT INDEXING ALONG OTHER DIMENSIONS

51 N2=N2+1  
  DO 54 J=1,MM1  
    I(J)=I(J)+<2  
    IF(I(J).LT.N) GO TO 54  
    I(J)=I(J)-N

54 CONTINUE  
  GO TO 30

52 N3=N3+1  
  I(1)=<3\*N3+<4\*N4  
  IF(I(1).LT.N) GO TO 21  
  I(1)=I(1)-N  
  GU TO 21

53 N4=N4+1  
  I(1)=<4\*N4  
  GU TO 21

C UNSCRAMBLING TRANSFORM RESULTS

1000 II=1  
  J=1  
  GU TO 1001

1002 IF(J.GT.N) GO TO 1003  
  II=II+KOUT

1004 IF(II.LE.N) GO TO 1001  
  II=II-1  
  GU TO 1004

1001 A(J)=XR(II)  
  B(J)=-XI(II)  
  J=J+1  
  GU TO 1002

C 2 POINT TRANSFORM

200 URX=UR(1)+UR(2)  
  UIX=UI(1)+UI(2)  
  UR(2)=UR(1)-UR(2)  
  UI(2)=UI(1)-UI(2)  
  UR(1)=URX  
  UI(1)=UIX  
  GU TO 40

C 3 POINT TRANSFORM

300 AR=UR(2)+UR(3)  
  AI=UI(2)+UI(3)  
  UR1=-1.5D0\*AR

$\alpha_{11} = -1.500000$   
 $\alpha_{12} = 0.8660254050000*(UR(2)-UR(3))$   
 $\alpha_{13} = 0.8660254050000*(UR(2)+UR(3))$   
 $UR(1) = AR + UR(1)$   
 $UI(1) = A11 + UI(1)$   
 $UR(1) = UI(1) + UR(1)$   
 $UI(1) = UI(1) + A11$   
 $UR(2) = AR1 - A12$   
 $UI(2) = A11 + A12$   
 $UR(3) = A11 + A12$   
 $UI(3) = A11 - A12$   
 $UR(4) = 0.70 - 40$

#### C 4 POINT TRANSFORM

$AR1 = UR(1) + UR(3)$   
 $A11 = UI(1) + UI(3)$   
 $AR2 = UR(1) - UR(3)$   
 $A12 = UI(1) - UI(3)$   
 $AR3 = UR(2) + UR(4)$   
 $A13 = UI(2) + UI(4)$   
 $AR4 = UR(2) - UR(4)$   
 $A14 = UI(2) - UI(4)$   
 $UR(1) = AR1 + AR3$   
 $UI(1) = A11 + A13$   
 $UR(2) = AR2 - A14$   
 $UI(2) = A12 + AR4$   
 $UR(3) = AR1 - AR3$   
 $UI(3) = A11 - A13$   
 $UR(4) = AR2 + A14$   
 $UI(4) = A12 - AR4$   
 $UR(5) = 0.70 - 40$

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#### C 5 POINT TRANSFORM

$AR1 = UR(2) + UR(5)$   
 $A11 = UI(2) + UI(5)$   
 $AR2 = UR(2) - UR(5)$   
 $A12 = UI(2) - UI(5)$   
 $AR3 = UR(3) + UR(4)$   
 $A13 = UI(3) + UI(4)$   
 $AR4 = UR(3) - UR(4)$   
 $A14 = UI(3) - UI(4)$   
 $AR5 = AR1 + AR3$   
 $A15 = A11 + A13$   
 $UR1 = 0.951056516300*(AR2+AR4)$   
 $UI1 = 0.951055516300*(A12+A14)$   
 $UR2 = 1.538841764000*AR2$   
 $UI2 = 1.538841759000*A12$   
 $UR3 = 0.363271264000*AR4$   
 $UI3 = 0.363271264000*A14$   
 $UR4 = 0.559016494400*(AR1+AR3)$   
 $UI4 = 0.559016494400*(A11+A13)$   
 $UR5 = -1.250000*AR5$   
 $UI5 = -1.250000*A15$   
 $UR(1) = UR(1) + AR5$   
 $UI(1) = UI(1) + A15$   
 $UR(2) = UR(1) + AR5$   
 $UI(2) = UI(1) + A15$   
 $AR1 = AR5 + AR4$   
 $A11 = A15 + A14$   
 $AR2 = AR5 - AR4$   
 $A12 = A15 - A14$

$A = 3 \times M2 - M3$

$M13 = M11 + M13$

$AR4 = AR1 - MR2$

$A11 = M11 - M12$

$MR2 = AR1 - A13$

$M11(2) = M11 + A13$

$MR(3) = MR2 + A14$

$M11(3) = A12 - AR4$

$MR(4) = AR2 - A14$

$M11(4) = A12 + AR4$

$MR(5) = AR1 + A13$

$M11(5) = A11 - AR3$

GO TO 40

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### C 7 POINT TRANSFORM

$MR1 = MR(2) + MR(7)$   
 $A11 = M11(2) + M11(7)$   
 $AR2 = MR(2) - MR(7)$   
 $A12 = M11(2) - M11(7)$   
 $AR3 = MR(3) + MR(5)$   
 $A13 = M11(3) + M11(6)$   
 $AR4 = MR(3) - MR(6)$   
 $A14 = M11(3) - M11(5)$   
 $AR5 = MR(4) + MR(5)$   
 $A15 = M11(4) + M11(5)$   
 $AR6 = MR(4) - MR(5)$   
 $A16 = M11(4) - M11(5)$   
 $AR7 = AR1 + AR3 + AR5$   
 $A17 = M11 + A13 + A15$   
 $MR1 = -1.16666666700 * AR7$   
 $M11 = -1.16666666700 * A17$   
 $MR2 = 0.790156468800 * (AR1 - AR5)$   
 $M12 = 0.790155468800 * (A11 - A15)$   
 $MR3 = 0.05585426800 * (AR5 - AR3)$   
 $M13 = 0.05585426800 * (A15 - A13)$   
 $MR4 = 0.73430220100 * (AR3 - AR1)$   
 $M14 = 0.73430220100 * (A13 - A11)$   
 $MR5 = 0.44045855200 * (AR2 + AR4 - AR6)$   
 $M15 = 0.44045855200 * (A12 + A14 - A16)$   
 $MR6 = 0.34087243100 * (AR2 + AR6)$   
 $M16 = 0.34087243100 * (A12 + A16)$   
 $MR7 = -0.53346436100 * (-AR6 - AR4)$   
 $M17 = -0.53346436100 * (-A16 - A14)$   
 $MR8 = 0.87484224100 * (AR4 - AR2)$   
 $M18 = 0.87484224100 * (A14 - A12)$   
 $M1(1) = M2(1) + AR7$   
 $M1(1) = M11(1) + A17$   
 $AR1 = MR(1) + M11$   
 $A11 = M11(1) + M11$   
 $AR2 = AR1 + MR2 + AR3$   
 $A12 = M11 + M12 + M13$   
 $AR3 = AR1 - MR2 - MR4$   
 $A13 = M11 - M12 - M14$   
 $AR4 = AR1 - MR3 + MR4$   
 $A14 = M11 - M13 + M14$   
 $AR5 = MR5 + MR6 + MR7$   
 $A15 = M15 + M16 + M17$   
 $AR6 = MR5 - MR6 - MR8$   
 $A16 = M15 - M16 - M18$

$\Delta R(7) = MR(5) - MR(7 + MR(8))$   
 $\Delta R(7) = 111 - 117 + MR(8)$   
 $\Delta I(2) = \Delta R(2) - A15$   
 $\Delta I(2) = A12 + A15$   
 $\Delta I(3) = A13 - A15$   
 $\Delta I(3) = A13 + A15$   
 $\Delta R(4) = A14 + A17$   
 $\Delta I(4) = A14 - A17$   
 $\Delta R(5) = A14 - A17$   
 $\Delta I(5) = A14 + A17$   
 $\Delta R(6) = A13 + A16$   
 $\Delta I(6) = A13 - A16$   
 $\Delta R(7) = A12 + A15$   
 $\Delta I(7) = A12 - A15$   
GO TO 40

#### C - PRESENT TRANSFORM

$MR(1) = DR(2) - DR(8)$   
 $A11 = II(2) - II(8)$   
 $\Delta R(2) = DR(2) + DR(8)$   
 $\Delta I(2) = II(2) + II(8)$   
 $\Delta R(3) = DR(4) - DR(6)$   
 $\Delta I(3) = II(4) - II(6)$   
 $\Delta R(4) = DR(4) + DR(6)$   
 $\Delta I(4) = II(4) + II(6)$   
 $\Delta R(5) = DR(1) - DR(5)$   
 $\Delta I(5) = II(1) - II(5)$   
 $\Delta R(6) = DR(1) + DR(5)$   
 $\Delta I(6) = II(1) + II(5)$   
 $\Delta R(7) = DR(3) - DR(7)$   
 $\Delta I(7) = II(3) - II(7)$   
 $\Delta R(8) = DR(3) + DR(7)$   
 $\Delta I(8) = II(3) + II(7)$   
 $MR1 = 0.707106781200 * (AR1 + AR3)$   
 $MR1 = 0.707105781200 * (AI1 + AI3)$   
 $MR2 = 0.707106781200 * (AR2 - AR4)$   
 $MR2 = 0.707105781200 * (AI2 - AI4)$   
 $MR3 = AR2 + AR4$   
 $MR3 = A12 + A14$   
 $MR4 = AR6 + AR8$   
 $MR4 = A16 + A18$   
 $MR5 = AR6 - AR8$   
 $MR5 = A16 - A18$   
 $MR6 = AR1 - AR3$   
 $MR6 = A11 - A13$   
 $MR7 = AR5 + MR2$   
 $MR7 = A15 + A12$   
 $MR8 = AR5 - MR2$   
 $MR8 = A15 - A12$   
 $MR9 = AR7 + MR1$   
 $MR9 = A17 + II1$   
 $MR10 = AR7 - MR1$   
 $MR10 = A17 - II1$   
 $DR(1) = MR4 + MR3$   
 $II(1) = II4 + II3$   
 $DR(2) = DR7 - DR9$   
 $II(2) = II7 + II9$   
 $DR(3) = DR5 - DR6$   
 $II(3) = II15 + II6$   
 $DR(4) = DR8 + II10$

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II(4)=II(4)+II(10)  
II(5)=II(5)-II(2)  
II(5)=II(5)-II(3)  
II(6)=II(6)-II(10)  
II(6)=II(8)+II(10)  
II(7)=II(5)+II(6)  
II(8)=II(7)+II(9)  
II(9)=II(7)-II(8)  
G= TD - 40

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G = 4 POINT TRANSFORM

AI1=II(2)+II(9)  
AI1=II(2)+II(9)  
AI2=II(2)-II(9)  
AI2=II(2)-II(9)  
AI3=II(3)+II(8)  
AI3=II(3)+II(8)  
AI4=II(3)-II(8)  
AI4=II(3)-II(8)  
AI5=II(5)+II(6)  
AI5=II(5)+II(6)  
AI6=II(5)-II(6)  
AI6=II(5)-II(6)  
AI7=II(4)+II(7)  
AI7=II(4)+II(7)  
AI8=II(4)-II(7)  
AI8=II(4)-II(7)  
AK=AK1+AK3+AK5  
AI=AI1+AI3+AI5  
MR1=-0.500\*AK7  
M11=-0.500\*AI7  
MR2=0.8660254038D0\*AK8  
M12=0.8660254038D0\*AI8  
MR3=0.19746542D0\*(-AK1+AK5)  
M13=0.19746542D0\*(-AI1+AI5)  
MR4=0.56857902D0\*(AK1-AK3)  
M14=0.56857902D0\*(AI1-AI3)  
MR5=0.3711136D0\*(-AK3+AK5)  
M15=0.3711136D0\*(-AI3+AI5)  
MR6=0.54253179D0\*(AK2-AK6)  
M16=0.54253179D0\*(AI2-AI6)  
MR7=0.10025579D0\*(AK2+AK4)  
M17=0.10025579D0\*(AI2+AI4)  
MR8=0.44227547D0\*(-AK4-AK6)  
M18=0.44227547D0\*(-AI4-AI6)  
MR9=-1.5D0\*AK4  
M19=-1.5D0\*AI4  
MR10=0.8660254038D0\*(AK2-AK4+AK6)  
M11=0.8660254038D0\*(AI2-AI4+AI6)  
AK1=II(1)+AK3  
AI1=II(1)+II1  
II(1)=AI+AI7+II(1)  
AK=II(1)+II9  
AI=II(1)+II9  
AI2=II14-II15  
AK3=AK3+II24

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C 16 PRESENT TRANSFORM  
 1600 AR1=UR(1)+UR(4)  
 AR2=U1(1)+U1(4)  
 AR3=UR(5)+UR(13)  
 AR4=U1(5)+U1(13)  
 AR5=UR(3)+UR(11)  
 AR6=U1(3)+U1(11)  
 AR7=UR(7)+UR(15)  
 AR8=U1(7)+U1(15)  
 AR9=UR(7)+UR(15)  
 AR10=U1(7)+U1(15)  
 AR11=UR(2)+UR(10)  
 AR12=U1(2)+U1(10)  
 AR13=UR(2)+UR(10)  
 AR14=U1(2)+U1(10)  
 AR15=UR(4)+UR(12)  
 AR16=U1(4)+U1(12)  
 AR17=UR(4)+UR(12)  
 AR18=U1(4)+U1(12)  
 AR19=UR(6)+UR(14)  
 AR20=U1(6)+U1(14)

A<sub>1</sub>12=0.7071067812D0\*(A<sub>1</sub>14-A<sub>1</sub>21)  
A<sub>1</sub>13=0.7071067812D0\*(A<sub>1</sub>14+A<sub>1</sub>21)  
A<sub>1</sub>14=0.7071067812D0\*(A<sub>1</sub>14-A<sub>1</sub>21)  
A<sub>1</sub>15=A<sub>1</sub>14+A<sub>1</sub>21  
A<sub>1</sub>16=A<sub>1</sub>13+A<sub>1</sub>15  
A<sub>1</sub>17=A<sub>1</sub>15+A<sub>1</sub>16  
A<sub>1</sub>18=A<sub>1</sub>15+A<sub>1</sub>16  
A<sub>1</sub>19=A<sub>1</sub>17+A<sub>1</sub>11  
A<sub>1</sub>20=A<sub>1</sub>17-A<sub>1</sub>11  
A<sub>1</sub>21=A<sub>1</sub>17+A<sub>1</sub>11  
A<sub>1</sub>22=A<sub>1</sub>18+A<sub>1</sub>20  
A<sub>1</sub>23=A<sub>1</sub>18+A<sub>1</sub>14  
A<sub>1</sub>24=A<sub>1</sub>18-A<sub>1</sub>14  
A<sub>1</sub>25=A<sub>1</sub>10+A<sub>1</sub>12  
A<sub>1</sub>26=A<sub>1</sub>10-A<sub>1</sub>12  
A<sub>1</sub>27=A<sub>1</sub>12-A<sub>1</sub>10  
A<sub>1</sub>28=A<sub>1</sub>12+A<sub>1</sub>10  
A<sub>1</sub>29=A<sub>1</sub>14-A<sub>1</sub>13  
A<sub>1</sub>30=A<sub>1</sub>14+A<sub>1</sub>13  
A<sub>1</sub>31=0.7071067812D0\*(A<sub>1</sub>14-A<sub>1</sub>21)  
U<sub>1</sub>(1)=A<sub>1</sub>17+A<sub>1</sub>22  
U<sub>1</sub>(1)=A<sub>1</sub>17+A<sub>1</sub>22  
U<sub>1</sub>(9)=A<sub>1</sub>17-A<sub>1</sub>22  
U<sub>1</sub>(9)=A<sub>1</sub>17-A<sub>1</sub>22  
A<sub>2</sub>29=A<sub>1</sub>15-A<sub>1</sub>16  
A<sub>1</sub>24=A<sub>1</sub>15-A<sub>1</sub>16  
A<sub>1</sub>30=A<sub>1</sub>11-A<sub>1</sub>2  
A<sub>1</sub>30=A<sub>1</sub>11-A<sub>1</sub>2  
m<sub>1</sub>=0.7071067812D0\*(A<sub>1</sub>14-A<sub>1</sub>21)  
m<sub>1</sub>=0.7071067812D0\*(A<sub>1</sub>14-A<sub>1</sub>21)  
m<sub>2</sub>=0.7071067812D0\*(A<sub>1</sub>14-A<sub>1</sub>21)  
m<sub>2</sub>=0.7071067812D0\*(A<sub>1</sub>14-A<sub>1</sub>21)  
m<sub>3</sub>=0.3428834324D0\*(A<sub>1</sub>22+A<sub>1</sub>26)  
m<sub>3</sub>=0.3428834324D0\*(A<sub>1</sub>22+A<sub>1</sub>26)  
m<sub>4</sub>=0.3069829651D0\*A<sub>2</sub>24  
m<sub>4</sub>=0.3069829651D0\*A<sub>2</sub>24  
m<sub>5</sub>=-0.5411961001D0\*A<sub>2</sub>26  
m<sub>5</sub>=-0.5411961001D0\*A<sub>2</sub>26  
A<sub>1</sub>32=A<sub>1</sub>18-A<sub>1</sub>20  
A<sub>1</sub>32=-A<sub>1</sub>18+A<sub>1</sub>20  
A<sub>1</sub>33=A<sub>1</sub>2-A<sub>1</sub>5  
A<sub>1</sub>33=-A<sub>1</sub>2+A<sub>1</sub>5  
A<sub>1</sub>34=U<sub>1</sub>(5)-U<sub>1</sub>(13)  
A<sub>1</sub>34=-U<sub>1</sub>(5)+U<sub>1</sub>(13)  
m<sub>6</sub>=0.7071067812D0\*(A<sub>1</sub>14-A<sub>1</sub>21)  
m<sub>6</sub>=0.7071067812D0\*(A<sub>1</sub>14-A<sub>1</sub>21)

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01(1)=1124-1214  
01(1)=1208-129  
01(1)=1208-129  
01(1)=1220-1220  
01(1)=1224-1228  
01(1)=1228-114  
01(1)=1232-1216  
01(1)=1236-1212  
01(1)=1240-1220  
01(1)=1244-1232  
01(1)=1248-1232  
01(1)=1252-1230  
01(1)=1256-1230  
01(1)=1260-113  
01(1)=1264-1218  
01(1)=1268-1227  
01(1)=1272-1227  
GO TO 40

1003 RETIRE

END

ROUTINE: PDATAUT

REALS C12,C2,A,B,CDE,PHB

COMMON/DFT/A(5040),B(5040)

COMMON/ZRAZ2/NE,X3,C12

COMMON,T

I=ITER, START, STOP

S=1,NE,1,0,I=NE,2

DEG=57.29576

IF(X3,NE,315) GO TO 101

C12=A(1)\*\*2+B(1)\*\*2

START=1

STOP=18

GO TO 200

101 IF(X3,NE,314) GO TO 102

START=19

STOP=31

GO TO 200

102 IF(X3,NE,313) GO TO 103

START=32

STOP=40

GO TO 200

103 IF(X3,NE,312) GO TO 104

START=41

STOP=47

GO TO 200

104 IF(X3,NE,311) GO TO 105

START=48

STOP=54

GO TO 200

105 IF(X3,NE,310) GO TO 80

START=55

STOP=59

200 DO 50 I=START,STOP

S=(I-1)\*17\*(N\*T)

PHA=PDATAUT(I,17\*(N\*T))

IF(S,LT,0.0) PHA=PHA+180.0

IF(S,GT,1.0) GO TO 60

S+=1.0-S\*S

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THE SALT (S2)  
A STATEMENT  
AS IT APPEARS  
GIVEN BY JAMES E. DODD  
ON 12 AUGUST 1941 (AMERICAN) / (CONTINUED)  
CHIEF, 1000 LOGIC (62/612)  
OF THE 701 AND 703 PAA  
ON 10 AUGUST  
1941  
TO THE ATTACHE, FEDERAL BUREAU OF INVESTIGATION  
RE TEL

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